EXISTENCE OF FINITELY MANY EQUILIBRIUM SOLUTION BRANCHES AND HYSTERESIS LOOPS IN FERROELECTRIC MATERIALS

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INTRODUCTION

In the study of soft matter systems, there are many materials exhibiting ferroelectricity which refers to the permanent or spontaneous polarization. Due to the appearance of the polarization, the structures of these materials are so complicated that a great deal of effort has been invested from the viewpoints of both mathematics and physics. A various different patterns of the polarization have been observed in the physics literature. There are two typical types among them which are called ferroelectric and antiferroelectric phases. In the ferroelectric phase, the polarization prefers to be aligned to one direction so that the net polarization is not zero. But if the material in the antiferroelectric phase, the net polarization is zero because two opposite neighboring polarizations are
canceled. If an electric field is applied in such phases, the dipoles which are already pointed in the direction of the applied field will remain so aligned, but those which are oriented in the opposite direction to the field will tend to reverse their orientation. This results into nucleations and domain walls, and a relation between polarization and the applied field, i.e., a hysteresis loop, which is the most important characteristic of ferroelectric materials. The problem then arises of how to specify the critical fields that can induce the switching and how to understand the effect of the polarization reversal process on the hysteresis loop during the switching by the applied field. In order to understand the process of the polarization reversal, we use the Landau-Devonshire model developed by Cao and Cross (1).

RESULTS

We proved existence of finitely many branches of equilibrium configurations of the energies by means of local and global bifurcation analysis. We established existence of finitely many local minimizers of the energies for a certain range of the net polarization, and showed existence of finitely many nested hysteresis loops showing finer structures of ferroelectric materials. We obtained critical fields which can induce nucleations in terms of the polarization and the net polarization during the polarization reversal process. We also presented examples for applications of present analysis to other fields.

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